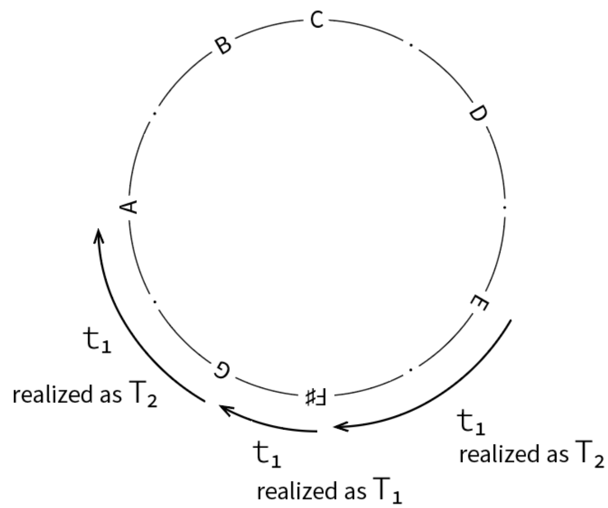
Recombinant melodies by Schubert and Yoshimatsu: Perspectives from solfége set theory

Select figures from



For transpositions in the diatonic scale, a generic 2nd can be a major or minor 2nd. The specific T_n counts semitones, and the generic t_n counts steps.

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Some of the u-of-v constructions, where ***u of v = u + v = w***

The written G4 of a bass clarinet in Bb is a concert F3 ***writtenPitch + instrumentPitch = concertPitch*** G4 + Bb2 = F3 *7 + −14 = −7*	***^2 in the key of E is F#*** ***deg + key = pc*** *^2 + E = F#* *2 + 4 = 6*
V of II is VI (of I) ***appliedRoot + tonicizedRoot = actualRoot*** *V + II = VI* *7 + 2 = 9*	***position re in 1# is A*** ***pos + sig = pc*** *re + 1# = A* *2 + 7 = 9*
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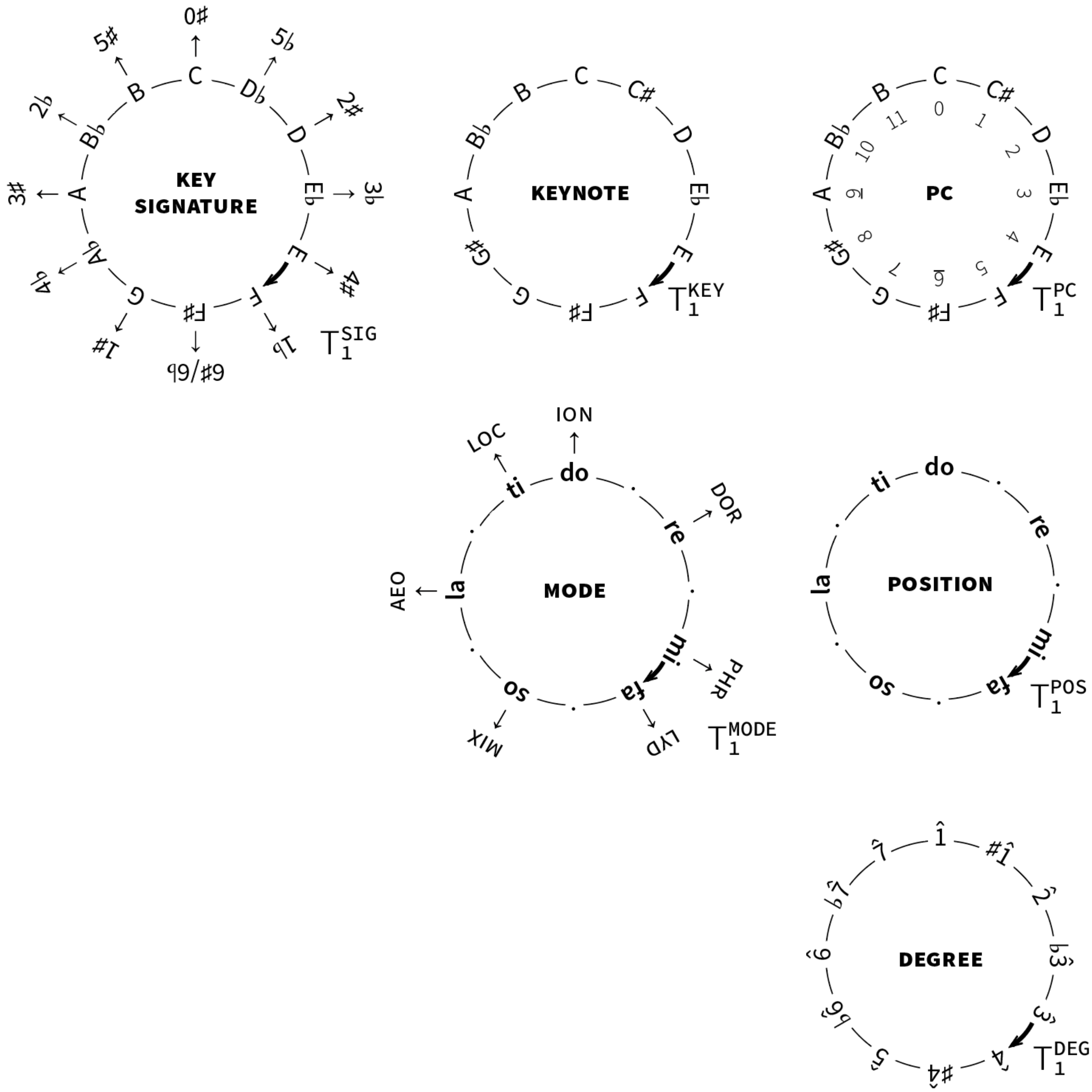

The six spaces of diatonic modality.

A single note in a key/mode context has six parameters with three degrees of freedom

(***sig, key, mode, pc, pos, deg***)

E.g., **1# E AEO: F#-ti-^2**

contrained by the equations

sig + mode = key

sig + pos = pc

deg + key = pc

Dependent transposition in sig x mode x pos space

$$T_x^{sig} T_{x+y}^{key} T_y^{mode} T_{x+z}^{pc} T_z^{pos} T_{z-y}^{deg}$$

Corresponding *independent* transposition for transpositions inside the scale

$$T_x^{sig} t_{y\prime}^{mode} t_{z\prime}^{pos}$$

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The 8 types of SIC (Single non-trivial generic Interval Class) transpositions

identity	$T_0^{SIG} t_0^{MODE} t_0^{POS}$
diatonic	$T_0^{SIG} t_0^{MODE} t_3^{POS}$
rel. trans.	$T_0^{SIG} t_3^{MODE} t_3^{POS}$
rel. pivot	$T_0^{SIG} t_3^{MODE} t_0^{POS}$
schubert	$T_7^{SIG} t_0^{MODE} t_3^{POS}$
brahms	$T_7^{SIG} t_3^{MODE} t_0^{POS}$
par. inflect.	$T_7^{SIG} t_3^{MODE} t_3^{POS}$
exact	$T_7^{SIG} t_0^{MODE} t_0^{POS}$

SIC transpositions by
perfect fifth and generic fourth
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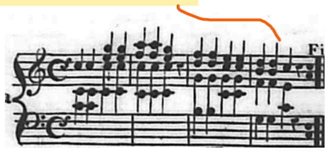
Two contrasting minor-mode variations

3♭ C min: C-**la**-^1

parallel inflection by 3 flats

$T_3^{sig} t_5^{mode} t_5^{pos}$

0♭ C maj: C-**do**-^1



0♭ A min: C-**do**- ^♭3

$T_0^{sig} t_5^{mode} t_0^{pos}$

relative pivot by a sixth

Georg Joseph Vogler, 16 Variations pour le clavecin ou pianoforte (pub. 1780?)

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Transposition of the last note into Variation X

theme	0♭	C	maj:	C	-	**do**	-	^1		
	T_3^{sig}	T_0^{key}	T_9^{mode}	T_0^{pc}		T_9^{pos}		T_0^{deg}		
Var. X	3♭	C	min:	C	-	**la**	-	^1		

parallel inflection by 3 flats

$T_3^{sig} t_5^{mode} t_5^{pos}$

realize mod-12

$t_0^{\square} \to T_0^{\square}$

$t_5^{\square} \to T_9^{\square}$

expand dependent elements by

$T_x^{sig} T_{x+y}^{key} T_y^{mode} T_{x+z}^{pc} T_z^{pos} T_{z-y}^{deg}$


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Transposition of the last note into Variation XIII

theme	0♭ C maj: C - **do** - ^1
	$T_0^{sig} T_9^{key} T_9^{mode} T_0^{pc} T_0^{pos} T_3^{deg}$
Var. X	0♭ A min: C - **do** - ^♭3

relative pivot
by a sixth

$T_0^{sig} t_5^{mode} t_0^{pos}$

realize mod-12

$t_0^{\square} \rightarrow T_0^{\square}$

$t_5^{\square} \rightarrow T_9^{\square}$

expand dependent
elements by

$T_x^{sig} T_{x+y}^{key} T_y^{mode} T_{x+z}^{pc} T_z^{pos} T_{z-y}^{deg}$